Deexcitation mechanisms in compound nucleus reactions

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1. Motivation

At the beginning of this course on de-excitation mechanisms in compound-nuclear reactions stands the result of an experiment \(^1\), which has been performed at GSI, Darmstadt. In this experiment, the full nuclide production after the bombardment of \(^{238}\text{U}\) by 1 GeV protons was recorded for the first time. For this purpose, \(^{238}\text{U}\) ions were accelerated to an energy of 238 GeV and impinged on a liquid hydrogen target. While the nuclear reaction is equivalent to the reaction of 1 GeV protons impinging on a \(^{238}\text{U}\) target, the advantage of inverting projectile and target lies in the fact that the heavy products of the reaction leave the target with high velocity and thus can be detected and identified with the help of a magnetic spectrometer and dedicated detectors.

Figure 1.1 shows that the nuclide distribution of this experiment covers many isotopes of practically all elements below uranium. Obviously, it consists of at least two rather well separated components, one peaking close to the projectile and another one located roughly between element 30 and 60. The residues shown in figure 1.1 are all produced by the decay of hot compound nuclei very close to \(^{238}\text{U}\), which are formed in the interaction of the proton with the heavy nucleus. Thus, the nuclide distribution reflects the features of the de-excitation process. It is the aim of this lecture to provide the knowledge which is necessary to understand the characteristics of this distribution on the basis of the different de-excitation channels involved.

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**Figure 1.1:** Measured nuclide cross section from the spallation of \(^{238}\text{U}\) by 1 GeV protons. The data for nuclei with 128 and 129 neutrons and for elements below \(Z = 7\) are missing due to technical limitations of the experiment.
2. Statistical model

The theoretical description of nuclear reactions with microscopic models with the full complexity of the interactions of the individual nucleons involved is a very challenging problem. Therefore, other approaches have been developed. One of those is the statistical model of nuclear reactions. It profits from concepts developed previously in the field of statistical mechanics. They have proven to be very powerful in explaining many features of nuclear reactions.

Some basic ideas of statistical mechanics are:

**Microstate:** A system is fully described by specifying the positions and the conjugate momenta of all its internal degrees of freedom (e.g. for a certain volume of a gas this refers to the positions and the momenta of all molecules). This defines the microstate of the system.

**Thermodynamical equilibrium:** A system is in thermodynamical equilibrium when each microstate occurs with the same probability. The assumption of equal a priori probabilities of all microstates lies at the very heart of statistical mechanics.

**Macrostate:** A macrostate is defined by specifying the external or global parameters of a system. This can e.g. be volume and total energy. For a many-particle system, there are generally a great number $\Omega$ of microstates, which are consistent with a given macrostate.

**Entropy:** The logarithm $S = \ln(\Omega)$ of the number of microstates, which are consistent with a given macrostate is named entropy. If the number of states of a system consisting of two subsystems is the product of the states of the subsystems, the entropy of the total system is the sum of the entropies of the subsystems.

**Temperature:** In thermodynamic equilibrium, all degrees of freedom carry the same energy $<\varepsilon>$ on the average. The global parameter $T = 2 <\varepsilon>$ is the temperature of the system.

The increase in entropy $S$ when the heat $dQ$ is adsorbed by a system at temperature $T$ is given by

$$dS = \frac{dQ}{T}$$

(2.1).

All these ideas appear in the statistical model of nuclear reactions. In particular, the statistical model of nuclear reactions assumes that the probabilities of different possible reaction channels are strongly determined by the corresponding numbers of available final states.
3. Nuclear properties

The nucleons in a nucleus are bound by the two-body nucleon-nucleon potential, which is attractive for large distances and repulsive for small distances as illustrated in figure 3.1. This is qualitatively similar to the interaction between the molecules of a liquid. Thus, the nucleus behaves like a liquid in several aspects. In particular, the nuclear radius $R$ grows with the mass number $A$ like $R = r_0 A^{1/3}$ with $r_0 = 1.16$ fm, and the density in the nuclear interior is constant, see figure 3.2.

![Figure 3.1: Qualitative illustration of the nucleon-nucleon potential.](image1)

![Figure 3.2: Nucleon density in various nuclei, deduced from electron scattering (B. Hahn et al., Phys. Rev. 101 (1956) 1131).](image2)

Like in a classical liquid, the potential in the nuclear interior is constant with a depth of about 40 MeV, which does not depend on the size of the nucleus.

However, the Fermionic nature of the nucleon implies some very specific quantum-mechanical features. According to the Thomas-Fermi approximation, each nucleon occupies a volume of size $\hbar^3/4$ in phase space in six-dimensional phase space formed by the 3 local and momentum coordinates. The factor $\frac{1}{4}$ reflects the degeneracy of the nucleon: spin up, spin down, and the two possibilities of isospin (neutron or proton). This implies that the volume of a nucleus in phase space is

$$V_{\text{ph}} = A \hbar^3 \frac{1}{4} = \frac{4}{3} \pi r_0^3 \frac{4}{3} \pi p_F^3 \frac{1}{4}$$

(3.1)
Minimum surface energy and minimum kinetic energy require spherical shape. Due to the constant density of nuclear matter the Fermi momentum $p_F$ (the radius of the Fermi sphere in momentum space) does not depend on mass number. Thus, the energy of the least bound nucleon above the bottom of the potential well (or equivalently the particle separation energy) in charge-symmetric nuclei with $N = Z$ does not depend on the mass of the nucleus, if the repulsive Coulomb interaction between protons is neglected. This would imply that the mass of particle-stable nuclei is not restricted. In fact, even the heaviest nuclei are stable against the emission of neutrons and protons. However, the stability of heavy nuclei is limited by fission and alpha decay.

There is another analogy between a nucleus and a liquid: Some nuclear excitation have a collective character, like rotations and vibrations of classical liquids. The most prominent collective excitations are quadrupole (and other multipole) vibrations of spherical nuclei, rotations of deformed nuclei and correlated oscillations of the neutrons and the protons of different kind. The latter are called giant resonances. Some are illustrated in figure 3.3. The most important one of these giant resonances is the isovector giant dipole resonance, where the proton subsystem and the neutron subsystem move along one direction with opposite phase.

**Figure 3.3:** Illustration of some giant resonances in nuclei with different spin (T) and isospin (I) components. The plus sign denotes protons, the minus sign neutrons.
4. Fermi-gas model

Differentiation of the nucleon number with respect to momentum in equation 3.1 yields the momentum distribution of the nucleons:

$$\frac{dn}{dp} = \frac{16}{3} \frac{n^2}{\hbar^3} r_0^3 p^2.$$  \hspace{1cm} (4.1)

Replacing $p^2 dp$ by $\sqrt{2m^3} \epsilon d\epsilon$ yields the density of single-particle states in the nuclear potential (see figure 4.1):

$$\frac{dn}{d\epsilon} = \frac{16}{3} \frac{n^2}{\hbar^3} r_0^3 \sqrt{2m^3} \sqrt{\epsilon}.$$  \hspace{1cm} (4.2)

(Consider that this result refers to a schematic potential hole with infinitely high walls. The result is somewhat different for a potential with finite depth or different shape, e.g. of Woods-Saxon type.)

![Figure 4.1: Density of occupied single-particle states in a cold and in an excited Fermionic system.](image)

The density of occupied single-particle states in an excited Fermionic system is given by the Fermi-Dirac statistics. It predicts the following occupation probability of the single-particle states of energy $\epsilon$:

$$P(\epsilon) = \frac{1}{1 + \exp((\epsilon - \mu) / T)}.$$  \hspace{1cm} (4.3)

The chemical potential $\mu$ is determined by the condition that the number of nucleons is preserved. Figure 4.1 shows that nucleons slightly below the Fermi energy are excited to levels slightly above the Fermi energy. It is plausible that the probability to excite a nucleon in a certain level decreases exponentially with its depth with respect to the Fermi energy, because excitations to levels below the Fermi energy are strongly suppressed due to the Pauli principle. Thus, the occupation probability is smeared out in a range of the order of the parameter $T$, which represents the nuclear temperature. Obviously, the levels deep in the potential well remain completely filled, and the corresponding nucleons remain essentially frozen.
5. Nuclear level density

The nuclear level density is defined as the number of nuclear states (microstates) per excitation-energy interval, realized as a specific pattern of single-particle excitations, at a given excitation energy. The most transparent and moreover exact method for evaluating the nuclear level density consists of counting the number of possible single-particle combinations with combinatorial methods. Figure 5.1 illustrates the method with a simple example. The number of possible excitations grows very fast with increasing excitation energy.

The fact that the slow increase of the single-particle level density of the Fermi-gas model is neglected is not crucial, because only the levels in the vicinity of the Fermi energy $\varepsilon_F$ are involved, where the level spacing varies only little.

![Figure 5.1: Determination of the nuclear level density with the combinatorial method. 20 nucleons are distributed on equidistant levels. An excitation energy of 5 times the level distance is assumed. Three of the seven possible single-particle excitations are shown. Option (d) violates the Pauli principle.](image)

There exist alternative approximate mathematical methods, e.g. the partition-function method. A simple analytical estimation of the energy dependence of the nuclear level density based on thermodynamics is given in the following:
While in an ideal gas all molecules take part in the thermal motion, the situation in a nucleus is quite different: As we have seen, thermal nuclear excitation only acts on the nucleons close to the Fermi surface in a range which is proportional to the nuclear temperature (equation 4.3). Summing up the excitation energies of the holes below the Fermi surface and the particles above the Fermi surface yields the excitation energy $E$ of the system, which grows like $E = a T^2$. (The average absolute energies of particles above the Fermi level and holes below the Fermi level grow linearly with temperature. The same is true for their numbers. The product leads to the quadratic dependence.) The constant is just named $a$. It is denoted as level-density parameter, as will be clear by equation 5.2 later.

The number of “microstates” in a nuclear system per energy interval (its level density) is given by all single-particle excitations, which are consistent with a certain excitation energy. We may estimate the energy dependence of the nuclear level density $\Omega$ by making use of equation (2.1):

$$d (\ln \Omega) = dS = dE/T = \sqrt{a E} dE$$

(5.1)

By integration one obtains:

$$S = 2 \sqrt{a E} \quad \text{or} \quad \Omega \propto e^{2 \sqrt{a E}}$$

(5.2)

Formula 5.2 is known as the expression of the Fermi-gas level density.

The deeper meaning is: In a nucleus, the number of nucleons sharing thermal excitations grows with temperature. Due to this variation of the effective number of degrees of freedom of the system the level density grows exponentially with the square root of the increasing excitation energy.

Only in a system, where the energy is proportional to the temperature, the exponential dependence of the level density vanishes. This implies that the effective number of degrees of freedom is independent of energy. This is the case, e.g., for an ideal gas, where every molecule has three translational degrees of freedom.
6. Shell effects

The assumption of the Thomas-Fermi approximation that each state occupies the same volume in phase space is generally only valid on average. An example, where this assumption is locally violated, is the 3-dimensional harmonic oscillator. The excitations at a certain energy can be realized as the sum of different excitations of the oscillator in the three directions in space. Thus, several single-particle states appear at the same energy, they are degenerate. This is illustrated in figures 6.1, where the single-particle scheme of a three-dimensional oscillator is compared with an infinitely deep hole and a hole with a finite depth. The sequence in the diffuse nuclear potential, including the spin-orbit interaction, is still different, but the non-uniformity of the single-particle level density is a general phenomenon.

The non-uniformity of the single-particle level sequence gives rise to shell effects, in the nuclear binding. It has also direct influence on the level density. This becomes obvious when applying the combinatorial method. In a magic nucleus, where a large single-particle energy gap coincides with the Fermi energy, a larger amount of excitation energy is needed for each single-particle excitation over the shell gap. Thus, the nuclear level density grows more slowly with increasing excitation energy.

This slower increase of the level density tends to gradually wash out the shell effect in the level density with increasing excitation energy. At nuclear temperatures much larger than the shell gap, the level density approaches the value of the Fermi-gas level density. However, the excitation energy must be counted above a fictive ground state, which is subject to a “backshift” in the order of the shell effect on the nuclear binding.
7. Pairing correlations

Nuclear binding energies show a systematic staggering: Nuclei with an even number of protons are more bound than nuclei with an odd number of protons. The same staggering appears as a function of neutron number. The physics behind this phenomenon has also consequences for the nuclear level density.

The Fermi-gas model as well as the nuclear shell model assume that the nucleons move independently in the nuclear potential as long as the nucleus is in its ground state. Collisions are forbidden, because there are no free states below the Fermi energy where the nucleons could be scattered to after the collision. Free states above the Fermi energy are not accessible due to energy conservation. However, this last argument is not true if the occupation function is somewhat smeared out, like it is in an excited nucleus, see figure 7.1. The even-odd effect in nuclear binding proves that scattering occurs.

The physics of nuclear pairing is very roughly described as follows:

1. Weakly coupled systems (interacting protons or neutrons) are described by perturbation theory: One state becomes particularly strongly bound compared to all the others.
2. Scattering can only occur if there are some free final states. This is only possible if the occupation function around the Fermi energy is smeared out.
3. In most nuclei at low excitation energy there is a net gain in binding energy by the pairing correlations, even if the energy due to the smearing of the occupation function is considered.
4. Fully paired states have angular momentum zero, because this provides maximum number of possibilities for scattering, e.g. by distributing two nucleons on +m and -m states.
5. Any unpaired nucleon cannot participate in the pairing correlation. In addition, it blocks a level which is thus unavailable for scattering by pairing. This explains the staggering in binding with even or odd number of protons or neutrons.
6. Excitations proceed by breaking pairs. They gradually destroy the pairing correlations.
7. The energy gain by pairing correlations is not limited to the ground state. It gradually decreases, mostly due to blocking and disappears at the critical energy.

The consequences for the level density are the following:

1. The nuclear binding energy is increased by a few MeV.
2. Below the critical energy (around 10 MeV), the nuclear level density grows less steeply than expected from the Fermi-gas formula. It is better approximated by a constant-temperature formula $\Omega \propto \exp^{E/T}$ with a constant value of the nuclear temperature $T$.
3. The energy dependence of the level density shows step-like structure, which correspond to successive pair breaking.
Strutinsky derived an analytical formula for the density of excited states $\Omega$ of a one-component system (neutrons or protons) on the basis of the combinatorial approach:\[2\]:

$$\Omega(E) = g^n (E - n\Delta)^{n-1} / [(n/2)! (n-1)!]$$ \hspace{1cm} (7.1)

where $g$ is the single-particle level density and $\Delta$ the pairing gap. This expression shows a step-like grow, whenever the excitation energy $E$ exceeds a multiple of the pairing-gap energy. A more complete formulation has been given by Ignatyuk and Sokolov [3].

The step-like structure of the level density has been observed only recently [4] as demonstrated by figure 7.1. Also the approximately linear dependence of $\ln(\Omega)$ as a function of excitation energy, which corresponds to the constant-temperature behaviour, is clearly seen.

In addition, one can observe that one needs first to break a pair for the first excitations in the even-even $^{116}$Sn nucleus: There is no level below about 1 MeV. In the odd-mass nucleus $^{117}$Sn the unpaired neutron can be excited to the available low-lying single-particle levels.

Figure 7.1: Experimentally determined level densities of $^{116}$Sn and $^{117}$Sn. Figure from ref. [4].
8. Collective excitations

The level density at low excitation energies, where spectroscopic information is available, is dominated by collective levels. Spherical nuclei like $^{76}$Se show vibrations, while deformed nuclei like $^{170}$Hf show rotational bands. According to the hybrid model of Bohr and Mottelson, each single-particle level can be the head of another vibrational sequence or a rotational band, respectively. This way, the level density predicted by the Fermi-gas expression is considerably increased. Vibrations may enhance the level density by up to a factor of 10, while rotations enhance the level density by almost 2 orders of magnitude. This enhancement should gradually vanish, because the ordered collective motion is damped at high excitation energies. Although this collective enhancement appears huge, it is not yet fully clear from experiment, up to which excitation energy it persists. Spectroscopic information is only available up to a few MeV above the yrast line.

$^{76}$Se

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<tr>
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<td>12$^*$</td>
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<tr>
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</tr>
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$^{170}$Hf

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</tr>
<tr>
<td>0.1</td>
<td>2$^*$</td>
</tr>
</tbody>
</table>

**Figure 8.1:** Typical collective excitation spectra build on the ground states of $^{76}$Se and $^{170}$Hg, respectively. The excitations of the spherical nucleus $^{76}$Se have vibrational character, while the deformed nucleus $^{170}$Hg shows a rotational band.
9. Compound nucleus

The compound-nucleus hypothesis of N. Bohr \(^5\) is probably the most often used concept for the description of nuclear reactions. Bohr divided a nuclear reaction in two steps, the formation of a compound nucleus and its decay. Apart from a few quantities, which are subject to conservation laws (energy, angular momentum, parity), the compound nucleus looses memory on the way it was formed. Thus, the two reaction steps can be considered as two separate steps, which proceed one after the other.

The compound-nucleus hypothesis was originally postulated for neutron-capture reactions at energies below a few tens of MeV, but it has proven to be valid in a much wider field. The main idea is that the compound nucleus is a quasi-bound system. Although the energy might be higher than the threshold of a certain de-excitation channel, e.g. the emission of a neutron, it is rather unprobable to concentrate the necessary energy to one degree of freedom. According to statistical arguments, the energy is distributed in approximately equal shares on all degrees of freedom most of the time. A quantitative estimate for the life time of a compound nucleus can be derived from the density of excited levels \(\Omega\) or the deduced mean level distance \(D = 1/\Omega\). By interference of different configurations with near-by energies, the wave function \(\varphi\) shows a periodicity in time:

\[
\varphi = \sum_{n=1}^{N} a_n \varphi_n \exp\left(\frac{-i E_n t}{\hbar}\right) = \exp\left(\frac{-i E_0 t}{\hbar}\right) \sum_{n=1}^{N} a_n \varphi_n \exp\left(\frac{-i n D t}{\hbar}\right) \quad (9.1)
\]

\[
\left|\varphi[t + \left(\frac{2 \pi \hbar}{D}\right)]\right|^2 = |\varphi(t)|^2 \quad (9.2)
\]

It follows that the wave function has a period of \(P = \frac{2 \pi \hbar}{D}\). The period \(P\) defines the time, the nucleus needs to realize a configuration that might lead to a specific decay.

This period has to be compared with the time \(\tau\) it takes for thermalisation, that means to distribute a disturbance, e.g. the energy of an incoming high-energetic particle, over all nucleons of the nucleus. The time \(\tau\) needed for thermalisation is in the order of the time, a nucleon is transported across the diameter of the nucleus by the Fermi motion. It turns out that \(P\) is long compared to \(\tau\), even at very high excitation energies. Thus, it is plausible to assume that the nucleus forgets how it was formed in a time of the order of \(\tau\). It explores all its possible configurations in the longer time scale \(P\).

Originally, the definition of the compound nucleus considered only the single-particle degrees of freedom. Later also the shape degree of freedom was included, e.g. deformation for fission and density for multifragmentation.

Mainly three possibilities have to be considered which violate the compound-nucleus hypothesis:

1. Direct reactions, in which only part of the degrees of freedom of the system are involved.
2. Fast processes, which proceed before formation of a compound nucleus (pre-equilibrium).
3. Slow dynamical evolution of shape, density or other global degrees of freedom, which create time-dependent global conditions for the intrinsic degrees of freedom. These reactions can be described by the methods of dissipative nuclear dynamics.
10. Evaporation

If the excitation energy of the compound nucleus is higher than the threshold of a certain decay channel, e.g. the emission of a neutron, there is a finite probability per time for this decay to happen. Weisskopf \(^6\) has given an estimation of the decay rate.

Weisskopf considered a nucleus B and a neutron enclosed in a volume \(V\). The neutron moves in a random way like a molecule of a gas with energy \(\epsilon\), corresponding to a velocity \(v = \sqrt{2\epsilon/m_n}\). If the cross section of the nucleus is denoted by \(\sigma\), the mean probability of the neutron being captured by the nucleus is

\[
P_c(\epsilon) = \sigma(\epsilon) v/V
\]  

(10.1)

From this, he derived the probability of the reverse process, the evaporation of a neutron from a nucleus A with excitation energy \(E_A\). (This is an application of the detailed-balance principle.) The probability \(P_n(\epsilon)\) for the emission of a neutron with an energy between \(\epsilon\) and \(\epsilon + d\epsilon\) is obtained by dividing the capture probability \(P_c(\epsilon)\) by the number of states \(\Omega_A(E_A)\) in which the neutron can be captured and by multiplying by the number of states \(\Omega_B(E_B)\) of the final nucleus and the number of states \(\Omega_n\) of the emitted neutron. The excitation energy \(E_B\) of the residual nucleus is given by \(E_B = E_A - S_n - \epsilon\) when \(S_n\) is the neutron separation energy of the compound nucleus.

The number of states of the neutron in volume \(V\) is

\[
\Omega_n = \frac{V g m_n}{2 \hbar^3/\pi} \sqrt{2 m_n \epsilon}
\]

(compare equation 4.2).

This yields:

\[
P_n(\epsilon) d\epsilon = P_c(\epsilon) \frac{\Omega_B(E_B)}{\Omega_A(E_A)} \Omega_n = P_c(\epsilon) \frac{\Omega_B(E_B)}{\Omega_A(E_A)} \frac{V g m_n}{2 \hbar^3/\pi} \sqrt{2 m_n \epsilon} d\epsilon = \sigma(\epsilon) \frac{g m_n \epsilon}{\hbar^3/\pi} \frac{\Omega_B(E_B)}{\Omega_A(E_A)} d\epsilon
\]

(10.2)

The energy-dependent factor \(\sigma(\epsilon) \cdot \epsilon \cdot \Omega_B(E_B) = \sigma(\epsilon) \cdot \epsilon \cdot \Omega_B(E_A - S_n - \epsilon)\) defines the shape of the energy spectrum of the emitted neutron. If \(\sigma(\epsilon)\) is set constant and \(\Omega_B\) is approximated by the constant-temperature formula, this is a Maxwell-Boltzmann distribution of the kind \(\epsilon \cdot \exp(-\epsilon/T)\) (see figure 10.1). The logarithmic slope of the energy spectrum in its high-energy tail is used to deduce the nuclear temperature.

![Figure 10.1: Shape of the Maxwell-Boltzmann distribution. The nuclear temperature was set to \(T = 1.5\ MeV\).](image-url)
Not only neutrons, but also protons and light nuclei can be evaporated from an excited nucleus. The emission probabilities are formulated in a similar way as for the neutron with a few modifications:

1. The final kinetic energy of the emitted charged particle is higher by the Coulomb barrier $B$.
2. The capture cross section $\sigma(\epsilon)$ is modified by an additional factor $(\epsilon - B) / \epsilon$ due to the deflection of the incoming particle in the Coulomb field of the nucleus.
3. Tunneling through the Coulomb barrier may be considered.
4. If the emitted nucleus has low-lying levels, these must be taken into account when calculating the number of final states.

The nucleus can also emit photons (gamma rays), but this decay branch is weak above the particle threshold. Most photons origin from the decay of the isovector giant dipole resonance (see figure 3.3), which is strongly coupled to the other excitation modes of the nucleus, like all giant resonances. Quadrupole radiation is emitted at the end of the de-excitation process, because it is finally the only way to remove the residual angular momentum at the end of the evaporation process.

If several de-excitation channels $i$ are in competition, their partial decay rates $P_i$ are obtained by integrating equation 10.2 (and the corresponding equations for charged-particle or gamma emission) over energy $P_i = \int_0^\infty P_i(\epsilon) d\epsilon$. Often, the decay probabilities are converted to decay widths $\Gamma_i = h P_i$, which have the dimension of energy.

Hauser and Feshach [7] developed an other approach, which considers the probability for the emission from one specific initial state to one specific final state. This way, e.g. angular-momentum dependencies can explicitly be taken into account.
11. Evaporation codes

Computer codes have been developed, which model the de-excitation of a compound nucleus. Most of them are “Monte-Carlo codes”. That means, they treat the decay of one excited nucleus after the other. Each time when the choice between different possibilities is open, their individual probabilities are calculated, and the decision for one of the possibilities is made at random with a random generator. Thus, the calculation resembles an experiment, where the reaction to be studied is repeatedly measured with a large number of projectiles. Many decay chains have to be calculated in order to obtain the required result with tolerable statistical fluctuations.

Figure 11.1 gives a schematic overview on the calculation procedure of an evaporation code. The originally formed compound nucleus has different decay channels open, evaporation of a neutron, different charged particles and emission of gamma rays. The relative decay probabilities are calculated, and a random choice is performed. As the next step, the kinetic energy of the emitted particle has to be determined. This is done by calculating the corresponding spectrum. A value from this continuous distribution is determined at random. With this result, the first daughter nucleus and its excitation energy are known. The above procedure is now repeated to determine the nature and the energy of the next particle emitted from the first daughter nucleus. The process stops when no decay is possible any more, that means when the final product is formed in its ground state.

![Figure 11.1: Calculation scheme of an evaporation code.](image)

The sequential character of the de-excitation process has important consequences. The evaporation process samples the nuclear properties step by step in restricted regions in excitation energy and nuclear composition and accumulates their outcome. Thus, the distribution of final residues reflects the influence of all de-excitation steps, e.g. all have to pass the region at low excitation energies, where shell effects and pairing correlations become strong.

It is expected that the experimental information provided at the end of the evaporation cascade shows the most strong fingerprints of the last stages of the de-excitation process, while the influence of previous stages and of the situation after the first reaction step are masked to a great extend. This is a severe problem for the study of the properties of hot nuclear matter, where one would like to know the properties of the products before the evaporation cascade.
12. General influence of excitation energy

Most often, several decay channels are in competition. The characteristic functional form of the level density \( \Omega \propto \exp(2\sqrt{a E}) \) has an important consequence for the excitation-energy dependence of this competition: Figure 12.1 demonstrates that the logarithms of the level densities approach each other with increasing energy. This means that the decay probabilities of the different channels come closer to each other. They asymptotically tend to become equal.

![Figure 12.1](image)

This means that decay channels, which are negligible at low excitation energies become more important or even comparable with the most important channels. This also implies that the number of open channels increases with excitation energy.
13. Evaporation corridor (attractor line)

The neutrons and protons are generally the most important evaporation channels. By only considering these two channels, one can explain already the most important feature of the evaporation process of highly excited nuclei: the phenomenon of the evaporation corridor.

Figure 13.1 shows the features of the occupied single-particle levels on different regions of the chart of the nuclides. On the beta-stability line, the Fermi levels of protons and neutrons are equal. However, for beta-stable excited nuclei the evaporation of neutrons prevails, because the emission of protons is hindered by the Coulomb barrier. Thus, the daughter nuclei become more and more neutron-deficient, and consequently the Fermi level of the protons increases, lowering the proton separation energy. At a certain point the decay widths $\Gamma_p$ and $\Gamma_n$ of protons and neutrons become comparable, and the evaporation of neutrons and protons comes to an asymptotic equilibrium. When starting from an even more proton-rich nucleus, the evaporation of protons would prevail, also approaching asymptotic equilibrium from the other side. Thus, there is a line, where all evaporation residues will finally end, if the evaporation cascade is long enough. This line is called attractor line [8], and the residue distribution in asymptotic equilibrium is called evaporation corridor.

![Figure 13.1: Schematic illustration of the evaporation attractor line (red dashed line). See text for details.](image)

The strong influence of the evaporation attractor line is illustrated in figure 13.2, where it can be seen that the evaporation process populates almost the same isotopes below tin for both projectiles, the neutron-deficient $^{124}$Xe and the neutron-rich $^{136}$Xe.
Figure 13.2: Measured nuclide distribution of the reactions $^{124}$Xe + Pb and $^{136}$Xe + Pb at 1 A GeV [$^\circ$]. Already for elements below $Z = 51$, the residues are located close to the evaporation attractor line in both reactions.
14. Fission

When fission was discovered by Hahn, Strassmann and Meitner, it was a surprise, because it had been overlooked so far that very heavy nuclei are not stable with respect to quadrupole oscillations. This effect makes a final end to the mass range of nuclei.

Figure 14.1 shows the evolution of the main deformation-dependent contributions to the binding energy: the surface energy and the Coulomb energy with elongation. While the surface energy favours the spherical shape, the Coulomb energy tends to increase the distances of the protons. This favours large elongations. Since the functional forms are different, the sum of these two energies creates a pocket. For nuclei which are not too heavy, a fission barrier stabilizes the spherical shape.

Thus, fission is an additional de-excitation channel to be considered in the de-excitation process of excited nuclei. The scheme of an evaporation code (figure 11.1) must be extended by another decay channel. Fission produces two fragments, which can be excited and therefore are the mother nuclei of two independent evaporation chains.
15. Transition-state model

The statistical model of nuclear reactions cannot readily be applied to nuclear fission, since the phase space of the end products has no influence on the reaction at the moment when the decision for fission is made.

Bohr and Wheeler [10] recognised that the fission probability is governed by the number of states above the fission barrier ("transition states") and not by the number of states of the final fragments. The configuration at the fission barrier is characterized by a minimum number of available levels on the way to fission. Therefore, it is the number of states in this configuration, which determines the probability for fission to occur.

The Bohr-Wheeler description of the fission decay width in a closed expression is very similar to the formula for particle decay:

$$\Gamma_f = \frac{1}{2\pi\Omega(E)} \int_0^{E-B_f} \rho_f(E) dE$$  \hspace{1cm} (15.1)

The picture is complicated by shell effects, which are responsible for the double-humped fission barrier.

The decision whether fission occurs is made at the outer barrier. In the transition-state model there is no return possible beyond this point.

![Figure 13.1: Schematic drawing of the potential energy on the fission path. The corresponding shapes are indicated below.](image)
16. Mass asymmetry

The transition-state model predicts only the fission-decay width. It does not specify the distribution of fission fragments in mass number and atomic number.

Figure 16.1 shows the major contributions to the deformation-dependent potential for a nucleus around $A = 200$, evaluated on the basis of the liquid-drop model.

Fig. 16.1: Surface and Coulomb energy at the fission barrier of a nucleus around $A = 200$ as a function of mass-asymmetric deformation.

The resultant height of the conditional saddle is shown in Figure 16.2. For light systems, the surface energy dominates, and asymmetric fission is favoured. At the so-called Businaro-Gallone [11] point, the potential is flat near symmetry. For heavy systems, the Coulomb energy dominates and favours symmetric mass splits. This gives a rough guidance for the mass distribution of the fission fragments: Fission-fragment mass distributions of light systems are U shaped, while mass distributions of heavy systems peak at symmetry.

Fig. 16.2: Height of the conditional saddle (the fission barrier under the condition of a specific mass-asymmetric deformation) for a light ($A \approx 50$), an intermediate ($A \approx 130$) and a heavy system ($A \approx 230$).
17. Fission channels

Fission of heavy nuclei at low excitation energy shows strong structural effects. Figure 17.1 reveals two types of structures in the element distribution for $^{226}\text{Th}$: On a fine scale there is an enhanced production of even-Z fragments. On a larger scale one can observe three bumps, which correspond to two fission components. One component favours symmetric splits and is behind the central peak. The other component favours mass-asymmetric splits and is responsible for the production of the two peaks near $Z = 36$ and $Z = 54$.

![Figure 17.1: Element distribution of the fission fragments of $^{226}\text{Th}$][12].

The calculation of the potential-energy shown in figure 17.2 demonstrates that shell effects in the potential are responsible for the gross structure of the measured $Z$ distribution.

![Figure 17.2: Calculated potential-energy landscape of $^{224}\text{Th}$ between fission barrier and scission. The calculation has been performed with the macroscopic-microscopic approach by V. V. Pashkevich (private communication). The parameter Alpha2 denotes the elongation, Alpha3 the mass asymmetric deformation of the system.](image)

The structural effects are not the same for different fissioning systems. It is still a challenge for theory to reproduce these data.
18. Application to the spallation of $^{238}$U

At the end of this lecture, we return to the first figure, the nuclide distribution resulting from the spallation of $^{238}$U by 1 GeV protons. It is shown again as figure 18.1 below.

![Figure 18.1: Measured nuclide cross section from the spallation of $^{238}$U by 1 GeV protons. The data for nuclei with 128 and 129 neutrons and for elements below Z=7 are missing due to technical limitations of the experiment.](image)

We are now able to understand the pattern on the chart of the nuclides, if we consider that the nuclei, which enter into the evaporation process, have only lost a few neutrons and protons with respect to $^{238}$U. Mostly due to the different impact parameters in the collision, they have a continuous distribution of excitation energies reaching up to a few 100 MeV. Nothing in the experimental distribution reminds the details of this initial stage.

The heavy component above $Z \approx 70$ is produced by evaporation of mostly neutrons and protons. It is directed to the neutron-deficient side versus the evaporation attractor line. This component dies out, because the initial excitation energy introduced by the 1 GeV proton is not sufficient to evaporate more than roughly 80 nucleons. The broad component around half the mass of $^{238}$U is produced by fission. It is stronger than the evaporation component, showing that the fission competition in the de-excitation stage is dominant. Fission occurring at low excitation energies is influenced by shell effects. This is seen e.g. in the production of particularly neutron-rich fragments close to $^{132}$Sn. Fission from higher excitation energies produces less neutron-rich fragments in a broad structureless distribution. Finally, the distribution reaches down to the lightest fragments due to very asymmetric mass splits. Although the corresponding emission barriers are high, this process becomes observable at very high excitation energies, where the emission probabilities of the different decay channels become similar.
For further reading:

Course on thermodynamics and statistical mechanics:
Richard Fitzpatrick: "Thermodynamics And Statistical Mechanics",
http://farside.ph.utexas.edu/teaching.html

Introduction into nuclear structure and nuclear reactions:

Classical text book:

Description of an early elaborate statistical deexcitation code:

General features of heavy-ion-induced compound-nuclear reactions around the Coulomb barrier:
T. Darrah Thomas, "Compound Nuclear Reactions Induced by Heavy Ions",
Annu. Rev. Nucl. Sci. 18 (1968) 343

Numerical methods for calculating the nuclear level density:

Direct experimental evidence for the influence of pair breaking in the nuclear level density:
U. Agvaanluvsan et al., "Evidence for the pair-breaking process in $^{116,117}$Sn",

Review on experimental results and theoretical models on spallation, fragmentation, fission and multifragmentation:
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5  N. Bohr, Nature 137 (1936) 344
6  V. F. Weisskopf, Phys. Rev. 52 (1937) 295
7  W. Hauser, H. Feshbach, Phys. Rev. 87 (1952) 366
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